University of Saskatchewan

Department of Computational Science
CMPT 260 ~~Midterm~~ Final
April 10, 1990

Marks

1. Express the following sentences in predicate calculus. Use the universe of discourse as indicated in parentheses.

3 a) There is no lion. (Animals In a Zoo)

3 b) Only Jim is a carpenter. (Employees in a company)

3 c) Cows give milk only if they have calfs, and if they are not fed well, they do not give much milk (Use cow(x), calf(x,y), fedwell(x), givemilk(x) and givemuchmilk(x))

2. Remove all quantifiers. State which symbols are variables, and which are constants. Use skolem functions where appropriate.

3 a) $\exists x\ P(x,y) \wedge \exists u\ \exists w\ Q(u,w)$.

3 b) $\forall x\ P(x,y) \vee \forall x\ \exists y\ Q(x,y)$

3 c) $(\forall x\ \exists y\ (M(x,y) \vee R(x))) \rightarrow \forall x\ P(x)$

10 3. Find the truth table for $((P \vee Q) \rightarrow R) \wedge (R \rightarrow P)$. Use this truth table to find the disjunctive normal form.

10 4. Consider the following premises:

$\exists x\ M(x)$

$\forall x\ (M(x) \rightarrow \exists y\ C(x,y))$

$\forall x\ (\exists y\ C(x,y) \rightarrow F(x))$

Prove, using natural derivation, that these premises lead one to the conclusion that there must be a y such that F(y) is true. State your premises, the rules employed to do your derivation, and the statements used for your derivation.

6 5. Write a Prolog program as follows. The pay rate for a marker depends on the year he is in, and these rates are stored in the database as facts of the form payrate(*year*, *rate*). For each student, there is a fact, containing the student name, the year he is in, and

the hours worked. These facts have the from student(*name, year, hoursworked*). Write the predicate "earned" of the form earned(*name, rate, amount*), where the amount is the appropriate rate, multiplied by the hours worked. A sample query could be, for instance, earned(smith, X, Y).

6 6) Some versions of prolog do not have the not predicate. Write the predicate notlast(*value, list*) for such a prolog version, where *value* a value, and the predicate is to succeed if this value is not the last member of *list*. Please note that <u>no</u> marks will be given when using the not predicate. You are, of course, allowed to use X \= Y to indicate that X and Y are not the same elements.

7 7) A data base contains a list of names in the form name(*firstname, lastname*). No two persons have the same *lastname*. Write a predicate fullname(X,Y), where X and Y are both lists, pertaining to the same individuals. However, whereas X contains only the last name, Y contains two elements for each individual, namely the first name, followed by the last name. You may assume that the database contains all names in question.

5 8) Let A, B and C be 3 sets.
Simplify $\sim((\sim A \cup \sim C) \cap B) \cup \sim(A \cup \sim(C \cap \sim B) \cup C)$

9) Let x P y be the relation that x is parent of y, let P~the converse of this relation.

4 a) The relations $P^\sim$, $P \circ P$, $P \circ P^\sim$ and $P \circ P \circ P^\sim \circ P^\sim$ all have names in a family context. Give the English names (In most cases, our language excludes the reflexive case. Ignore this fact for solving the problem. For instance, assume that every woman is her own sister.)

3 b) Let x R y be the relation that x and y have at least one ancestor in common. Define R in terms of P, P~ and their transitive closures.

12 10) The following program calculates the sum of the A[J] for J running from 1 to C.

```
J:=1;
Sum := A[J];
while J <> C-1 do
        J := J+1;
```

Sum := Sum + A[J]
od

a) Formulate the postcondition of this program.
b) Formulate the loop invariance.
c) Prove the correctness of the loop.
d) Use {P} while D do C od $\{P \wedge \neg D\}$ to prove that the program is correct. What are P and D in this concrete case? Be as specific as you can.
e) The program has a precondition. Identify this precondition.

6 11) Use resolution and prove that the following. Here, F means false. State which rules where used in each step.
$\neg P \vee Q \vee R, \neg P \vee \neg Q \vee R, P, \neg P \vee \neg R \vdash F$
(For those who forgot: Resolution means that if you have $A \vee B$, and $\neg A \vee C$, you can write $B \vee C$.}

12} Consider the following grammar. Here, S, B and C are nonterminals, and b, d, e are terminals.
1. $S \rightarrow bSB$
2. $S \rightarrow Ce$
3. $B \rightarrow d$
4. $C \rightarrow e$

4 a) Give the parsing function.
4 b) Give the trace (as in Table 6.3, page 242 handout) of the input string beed.

13) Let f be a mapping from A to B, where A = {a, 2, 4} and B = {2, 5, a, b}. Moreover, f(a) = 2, f(2} = 2 and f(4} = 5.

3 a) Find the domain, the codomain and the range of this function.
2 b) Is the function onto, into, or bijective?

14) Consider the Prolog statement
abc(X, a, C, B) :- de(X, X, B, C).

2 a) Express this statement in the form of predicate calculus.
3 b) Does the body of this statement unify with the head of a clause of the form de(X, C, a, Y). Justify your answer, and give the correspondence between varaibles and atoms.

105 **** THE END ****